Structural Damage Identification Using Virtual Flexibility Matrix Ⅰ: Simulation Verification

Tianguo Tang*[1], Xinsheng Xie[2]

[1] Civil and Structural department, School of Architecture and Environment, Sichuan University, Chengdu, 610065, China

[2] School of Water Resouce and Power Engineering, Sichuan University, Chengdu, 610065, China

ttgthy@sina.com; xxs6666@163.com

Received 31 May 2013; Accepted 26 September 2014; Published 13 March 2014

Abstract

The accident of modal characteristics directly provides an indication of structural damage. Based on changes in structural frequencies and mode shapes, a virtual flexibility matrix(VFM) technique is proposed in this paper to detect damage locations and their severity. The method is applied at an element level with a conventional finite-element model. The element damage index is established from the changes of the generalized strain in element's undamaged and damaged modes. The severity of the element's damage is also obtained simultaneously through the changeable value of the generalized strain. The method verified a number of damage scenarios from simulated truss models and simply supported beam and found the exact location and severity of damage. It has been demonstrated that using virtual flexibility matrix excluded the influences from the global mass and stiffness matrices in a structure and guaranteed successful identification for damage location. Numerically simulated results showed that the virtual flexibility matrix method could lead to satisfactory results in most cases.

Keywords

Damage; Structures; Modal Analysis; Virtual Flexibility Matrix (VFM); Identification

Introduction

A significant amount of research has been conducted in the area of nondestructive damage identification of structures. Many research studies on nondestructive detection of the damage location and estimate the severity of damage in a structure via changes in modal parameters have been performed. For example, for beams, attempts have been made to relate changes in natural frequencies to such influence as crack and local geometrical varies and to identify the damage location and magnitude from the measured or calculated vibration modes.

The nondestructive detection methods developed to date can be classed into four categories: (1) level 1 methods that only identify if damage has occurred; (2) level 2 methods that identify if damage has occurred and simultaneously determine the location of damage; (3) level 3 methods that identify if damage has occurred, determine the location of damage and estimate the severity of the damages; and (4) level 4 methods that identify damage has occurred, determine the location of damage, estimate the severity of the damage, and evaluation the impact of damage on the structure.

Despite these combined research efforts, several problems remain to be solved before damage identification in real structures becomes a routine activity. First of all, a need remains to improve the identification accuracy of damage localization and severity estimation by developing more robust expressions for damage indices which are highly sensitive to local damage on the structure. Secondly, a need remains to develop specialized theories of damage detection to simultaneously predict the location and estimate the severity of damage for different types of damages (e.g., different crack shapes and sizes, or corrosion, deterioration)in structures. Finally, a need remains to circumvent the reality that under the field condition only the lower modes in a large structure may be measurable.

The objective of this paper is to present, for truss

models and simply supported beam, a nondestructive evaluation method to locate damage and estimate the severity of damage, by using changes in modal parameters of the studied structures. This objective is accomplished in four tasks. First of all, the theoretical background of the virtual flexibility matrix algorithm for the structures is outlined. A damage-index method that simultaneously locates damage and estimates the size of the damages from the virtual flexibility matrix is summarized. Secondly, two truss models and a simply supported beam for which modal parameters of the lower three dynamic modes are computed from a finite element method are described, and structural damage can be represented by a decrease in the stiffness of the individual element. Thirdly, the proposed damage identification scheme and the collected modal data are used to localize damage and estimate severity of damage in the studied structure through searching for damaged element. Finally, the feasibility of the VFM algorithm is evaluated by quantifying the accuracy of identification results of the objective of structure. This work differs from other related attempts in at least two ways. At first, an attempt is made to develop a more robust damage index that removed the impacts from the numerical model (e.g. mass matrix) on identification results. Secondly, an attempt is made to simultaneously find the location of damage and estimate the severity of the damage.

The present method has been verified by a number of simulated damage scenarios for two truss models and a simply supported beam. Simulated data come from the FEM package ANSYS. All numerical results are obtained from routines developed in the MATLAB(1998) environment. It is demonstrated that the proposed damage identification technique can find the exact location and severity of damage for different simulated damage scenarios.

Theoretical Background

The VFM methodology that locates the damage and estimates severity of the damage directly from changes in modal characteristics of the target structure is presented in this paper. The modal characteristics of interest here are natural frequencies and mode shapes. Two sets of modal data are numerically simulated measured, one data set for the predamage structure and the other data set for the postdamaged structure. With these data, the method predicts damage location and estimates the severity of the damage.

Damage Identification Theory

Consider a structure of uniform cross section with NE members and N nodes. Assuming that the mode shapes that being normalized to the mass matrix and natural frequencies have been obtained theoretically, numerically, or experimentally. The flexibility matrix inversed from stiffness matrix is given by

$$F = K^{-1} = \Phi\Lambda^{-1}\Phi^T = \sum_{i=1}^{n}\frac{1}{\lambda_i}\varphi_i\varphi_i^T \qquad (1)$$

where F and K are the flexibility matrix and stiffness matrix, respectively; Φ is the mode shape matrix of the structure; $\Lambda = DIAG\{\lambda_i, i = 1,2,\cdots,n\}$ is the eigenvalue diagonal matrix; λ_i and φ_i are the ith eigenvalue and the ith mode shapes vector of the structure, respectively; and n is the total number of freedom degrees for the studied structure.

In reality, only the lower frequencies and mode shapes in a large complicated structure may be measurable. And then, the virtual flexibility matrix(VFM) is given by

$$F = K^{-1} = \tilde{F} == \sum_{i=1}^{m}\frac{1}{\lambda_i}\varphi_i\varphi_i^T \qquad (2)$$

where $\tilde{F}$ is the virtual flexibility matrix(VFM); m is the real number of obtained modes for the studied structure; and the other symbol are the same meaning as Eq. (1).

The $\tilde{F}$ for which constructed from modal data of the structure can be accomplished. The generalized displacement vector of the specialized type of structures(for example, beam) can be presented by

$$u = K^{-1}p = \tilde{F}p \qquad (3)$$

where p is the virtual load vector which not actual imposed on the studied structures,u is the generalized displacement vector. By integrating nodal displacement with the generalized displacement u of the node, the generalized strain can be computed.

Now, the generalized strain vector for which planar truss elements of the truss structures can be given by

$$\varepsilon_u^i = \frac{p_u^i}{A^i E_u^i},\ \varepsilon_d^i = \frac{p_d^i}{A^i E_d^i} \qquad (4)$$

where ε is axial strain of the truss elements; A and E are the transverse area and Young's modulus of the studied structure, respectively; Superscript i is the element number of the truss structure; Subscript u and d stand for predamaged and postdamaged conditions of the objective structure, respectively. Herein, p uses

unitary axial force for the planar truss element substitutes for the virtual load vector, see Fig.1(a). And so, the change ratio of axial strain for the truss element in the undamaged and the damaged structures is given by

$$G^i = \frac{\varepsilon_d^i - \varepsilon_u^i}{\varepsilon_u^i} = \frac{p_d^i E_u^i}{p_u^i E_d^i} - 1 \tag{5}$$

where G^i is the change ratio of axial strain due to the damage inflicted in ith element. In FEM work E_u value could obtain from material tests, and E_d value can be calculated by predamage percentages.

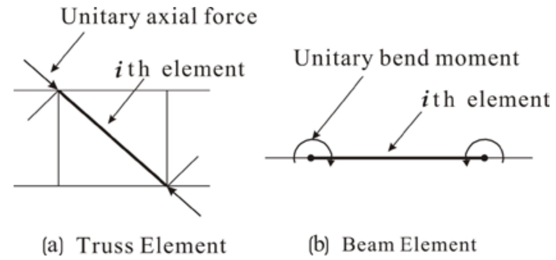


FIG.1 THE VIRTUAL LOAD SKETCH IN TWO TYPES OF ELEMENTS

Similarly, if the studied objective is beam-type structures, the flexual curvature can be presented by

$$K_u^i = \frac{M_u^i}{E^i I_u^i},\ K_d^i = \frac{M_d^i}{E^i I_d^i} \tag{6}$$

where K_u^i and K_d^i are the modal curvature of ith element in the undamaged and damaged structure, respectively; M_u^i and M_d^i substitute for the virtual load for which unitary bend moment is utilized in the ith beam element in the predamaged and postdamaged conditions, respectively, see Fig.1(b); I_u^i and I_d^i are the moment of inertia of ith element in the undamaged and damaged structure, respectively. The change ration of curvature can be presented by

$$G^i = \frac{K_d^i - K_u^i}{K_u^i} \tag{7}$$

where G^i is the change ratio of flexural curvature due to the damage influence on ith element.

Imposing Eq.(6) on Eq.(7) results in the following damage equation:

$$G^i = \frac{K_d^i - K_u^i}{K_u^i} = \frac{M_d^i I_u^i}{M_u^i I_d^i} - 1 \tag{8}$$

If the studied structure is determinate, the axial force p and bend moment M excluding influence from damage can be determined from the outside load imposed on the objective of structure. Now, we can define the severity index of damage as the following :

$$D^i = 1 - \frac{1}{1+G^i} \tag{9}$$

where D^i is the severity of damage in ith element for the objective of structures.

Up to now, the relation equations of damage location and the severity of the damage are derived.

To the determinate structure, the modes obtained from its modal tests are constructed; in addition, the flexibility matrix and the damaged index G and the severity of the damage D are simultaneously found by using Eq.(8) and Eq.(9). The change of G for the damaged element is nonzero value, and on the contrary, the value of G is zero for the undamaged element. But, the actual numerical stability or noise of the field tests both lead to the fluctuation of identification values for the damaged index G. Under these circumstances, the value of G may also be nonzero for the undamaged element. However, the value of G for the damaged element is much bigger than that for the undamaged element.

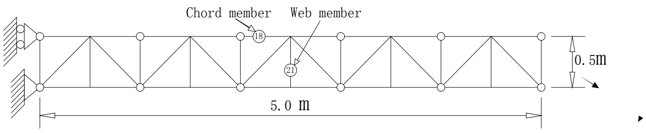
However, as for the statically indeterminate structures, its damaged parts would bring redistribution of internal force for the structure, and only the low modes can be obtained from field tests. Based on these modal data, the virtual flexibility matrix can be constructed by Eq.(2). Herein the change of G^i for the undamaged element is also nonzero value, but the change of G^i for the damaged element is much bigger than that for the undamaged element. So, the aforementioned method can be applied to detect damage for the statically indeterminate structures.

In order to apply the above method to the damage detection, a VFM procedure has been established and the successive damage identification scheme is presented. In this scheme, the VFM procedure starts with the initial analytical model through the reference measurements. In the frame of the discretization modeling, there are a large number of DOFs involved. In practice, it is impossible to measure all DOFs of a studied model due to the limited numbers of sensors. Therefore, mode shapes expansion is necessary. In this studied project, the expansion of modal mode shapes to unmeasured ones has been involved in the computational process.

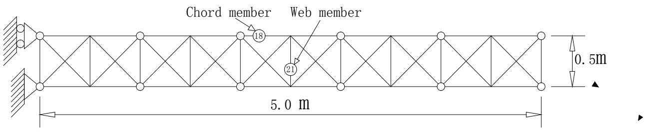
Simulation Verification—Plane Truss Model

Two plane truss structures, as shown in Fig.2, are firstly used to demonstrate the element damage index G in Eq.(8). The truss structure in Fig.2(a) is determinate and it is statically indeterminate structure

in Fig.2(b). The total numbers of elements are respectively 41 and 51. The material properties are all taken from Carbon Fiber Reinforcement Plastic(CFRP) where the elastic modulus E=42.4GPa and the density ρ=1,600 kg/m^3 and the area of cross-section for the truss element A=0.001 m^2. The two trusses may both be subjected to different simulated damage scenarios, i.e., various location and severity of damage summarized in Table 1, are conceived. Damage is simulated by reducing the stiffness of assumed elements. Three damage scenarios are considered(1) 10% reduction in bending stiffness at element 18;(2) 30% reduction in bending stiffness at element 21 and (3) both damages with 30% bending stiffness reduction at elements 18 and 21 designated in table 1. Similarly, Fig.2(b) shows another plane truss model including three damage cases.



(a) plane truss model



(b) plane truss model

FIG.2 SIMULATED TRUSS MODEL

For each scenarios, the structural dynamic characteristics (frequency and mode shapes) before and after the damage are calculated through the finite-element analysis package ANSYS. The modal data extracted from modal analysis will form the virtual flexibility matrix according to simulated results. The axial displacement of both nodes of for each truss element would be calculated simultaneously by analysis results form ANSYS package.

TABLE 1 SIMULATED DAMAGE SCENARIOS

TRUSS MODEL	DAMAGE CASES	DAMAGE ELEMENT	DAMAGE SEVERITY D
PLANE TRUSS MODEL (A)	CASE 1	18	10%
	CASE 2	21	30%
	CASE 3	18 AND 21	BOTH 30%
PLANE TRUSS MODEL (B)	CASE 1	22	10%
	CASE 2	26	30%
	CASE 3	22 AND 26	BOTH 50%

For each damage case, the damage location and severity estimation are performed as follows. Firstly, predamage and postdamage of the modal parameters of the first four modes (including frequencies and mode shapes) are obtained from modal analysis of the test truss model. Secondly, according to Eq.(2), the VFM can be built through using the aforementioned modal parameters of the first two modes. In step 3, assuming that each truss element is imposed a unitary axial virtual force, and the general axial displacement and the general strain of each node can be computed by Eq.(4) and (5), respectively. At last, the damage index $\boldsymbol{G}$ is obtained from the results from step 3 and the severity index $\boldsymbol{D}$ of damage is derived from Eq.(9). The results of the damage index method for the two damage scenarios using VFM measurement data are plotted in Fig.3 and Fig.4. The damage index G is separately estimated from two modes and four modes for comparison. It is also obvious that the first four modes is sufficient for reliable damage detection. Fig.5 shows that both damages for two elements could be localized at the points of changes in damaged element. It is demonstrated that the identification results of damage severity by lowering two modes have more errors than these obtained from lower four modes, as shown in Table 2. From the table, for severity index of damage $\boldsymbol{D}$, the errors from identification results are satisfactory and reasonable and the estimated severities are very close to the assumed damage sizes. The maximal error 5.94% occurs in case 3 of the truss model (b), which is subjected to a couple of damages. It's interpreted as follows: (1) the truss model is statically indeterminate structure in Fig.2(b) and (2) local damage of a element will bring the redistribution of axial stress for adjacent truss elements, on the contrary, the change of axial stress from the adjacent elements may have influence on identification results of the damaged element. Minimum error is 2.28% from plane truss model(b) in case 1, but both damages of two elements are on greater identification errors according to simulation analysis.

TABLE 2 DAMAGE SEVERITY IDENTIFICATION

Truss model		Plane truss model (a)			Plane truss model (b)		
Damage cases		Case 1	Case 2	Case 3	Case 1	Case 2	Case 3
Simulated damage Severity(G)		10%	30%	both 30%	10%	30%	both 50%
Using pre-two modes	Damage Severity(G) identification	13.04%	34.21%	33.86% 34.08%	12.28%	35.90%	55.27% 56.60%
	Errors(%)	3.04	4.21	3.97*	2.28	5.90	5.94*
Using pre-four modes	Damage Severity(G) identification	12.28%	33.33%	33.38% 33.55%	12.82%	35.06%	54.90% 56.05%
	Errors(%)	2.30	3.33	3.47	2.82	5.06*	5.48*

Note: superscript * is the averaged errors for a couple of damages

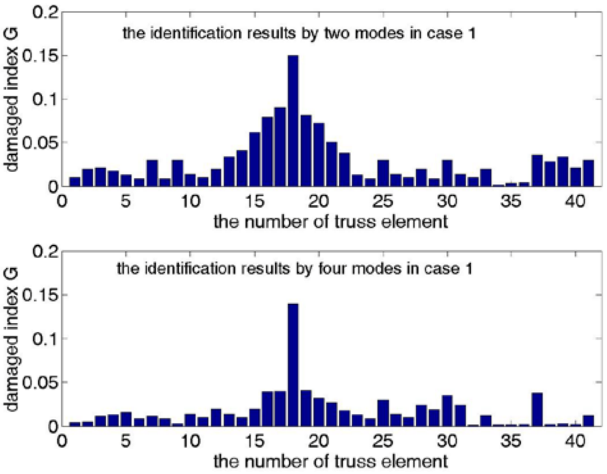


FIG.3 IDENTIFICATION RESULTS FOR TRUSS MODEL(A) IN CASE 1

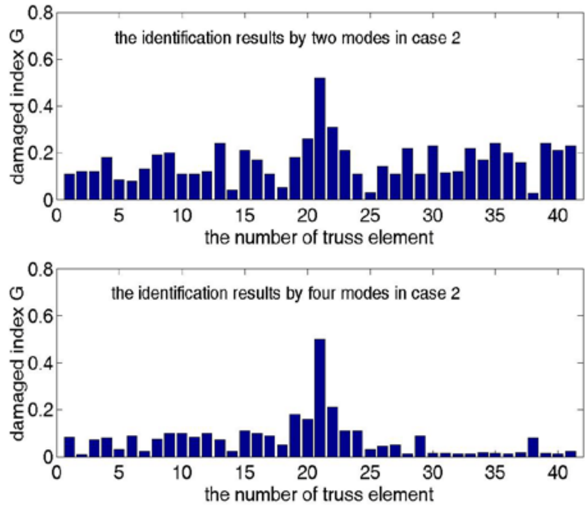


FIG.4 IDENTIFICATION RESULTS FOR TRUSS MODEL(A) IN CASE 2

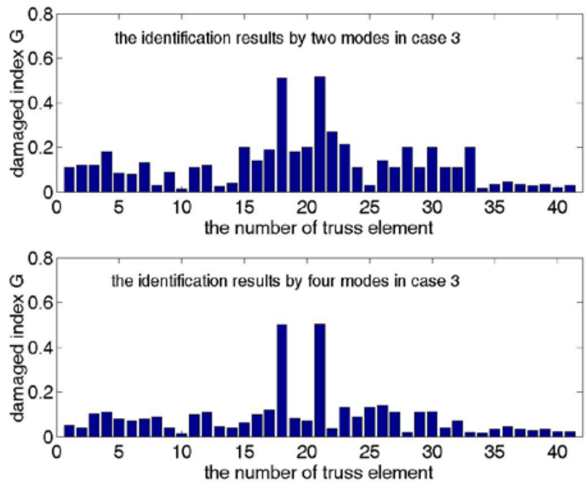


FIG.5 IDENTIFICATION RESULTS FOR TRUSS MODEL(A) IN CASE 3

Due to the determinate structure in Fig,2(a), the detection results of damage severity which are listed in Table.2, are more accurate than those from the statically indeterminate structure in Fig.2(b).

Simulation Verification—Simple Supported Beam

A simply supported beam model is used here to further verify the applicability of the proposed damage identification method. The geometrical cross-section size (H×B= 0.125 m×0.06 m) is conceived. The geometry and a section of the test beam are shown in Fig.6. The material properties are taken from homogeneous organic-glass, the Young's modulus of the test beam is 3.0 GPa and its density is 1,800kg/m^3. The element discreterization and damage scenarios are respectively shown in Fig.6 and Table.3. The simulated damage corresponds to a realistic pattern for a simple supported beam. The first four frequencies of both undamaged and damaged beams obtained from the finite-element method are listed in Table.3. Simultaneously, the first three damaged and undamaged mode shapes that are only vertical motion measured are listed in Fig.7~Fig.9.

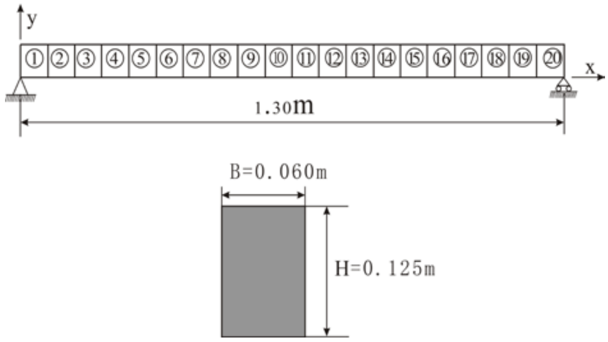


FIG.6 A SIMPLY SUPPORTED BEAM MODEL

TABLE.3 SIMULATED RESULTS OF THE SIMPLE SUPPORTED BEAM

Damage cases		0	1	2	3	4
Damage element		undamaged	10	10	5	5,10
Damaged Severity (G)		undamaged	30%	50%	50%	both 50%
Frequency /Hz	1th	70.67	66.48	66.66	67.22	68.45
	2nd	206.87	204.00	196.92	201.58	202.96
	3rd	462.43	455.29	453.61	456.32	456.36
	4th	849.26	841.64	839.54	828.46	826.47

The Euler-Bernoulli beam mode was selected as the damage detection model for the VFM technique. As described previously (i.e., as shown in Fig.6), the studied beam consists of a total of 20 beam elements of equal size. Modal parameters needed in the presented method are modal frequencies and mode shapes in predamage and postdamage structure. For individual mode shape, readings at the 21 nodal points from the FEM model were obtained.

The VFM can be constructed using the first three modal parameters and the general displacement of node to which the node's angle of rotation is extracted from each beam element is calculated through the

Finite Element Method. Then the damage location indices **G** were computed, as shown in Fig.10 and Fig.11. Fig.10 shows the identification results of damage index **G** for damaged case 1 and case 2. Fig.11 demonstrates the identification results of damage index **G** for damaged case 3 and case 4. In Fig.10, it is noted that the changes of damage index G are indicative to damage inflicted in the mid-span(element 10) in case 1 and case 2, as listed in Table 3. In this manner, Fig.11 note that changes of damage index respectively refer to a single damage inflicted in the left quarter-span (element 5) in case 3 and a couple of damages inflicted simultaneously in the mid-span (element 10) and the left quarter-span (element 5) in case 4, as shown in Table 3. The damage severity results for four cases are shown in Fig.12.

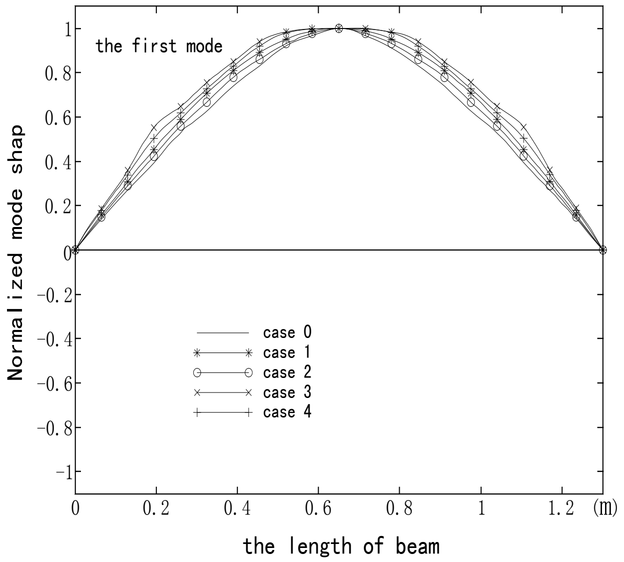


FIG.7 THE FIRST MODE SHAPE FOR SIMULATED BEAM

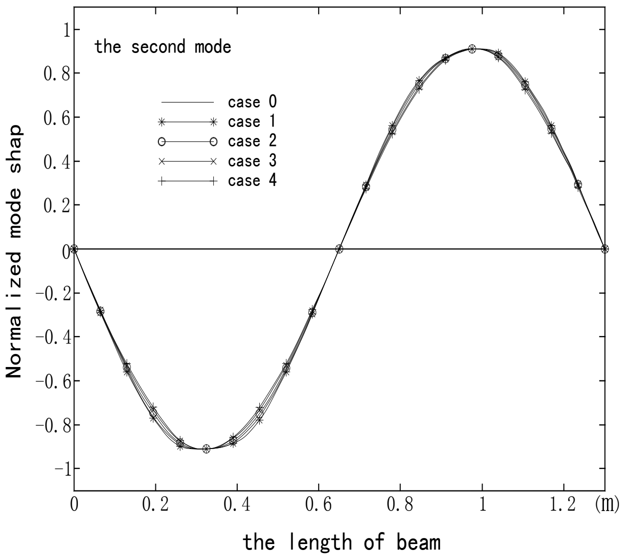


FIG.8 THE SECOND MODE SHAPE FOR SIMULATED BEAM

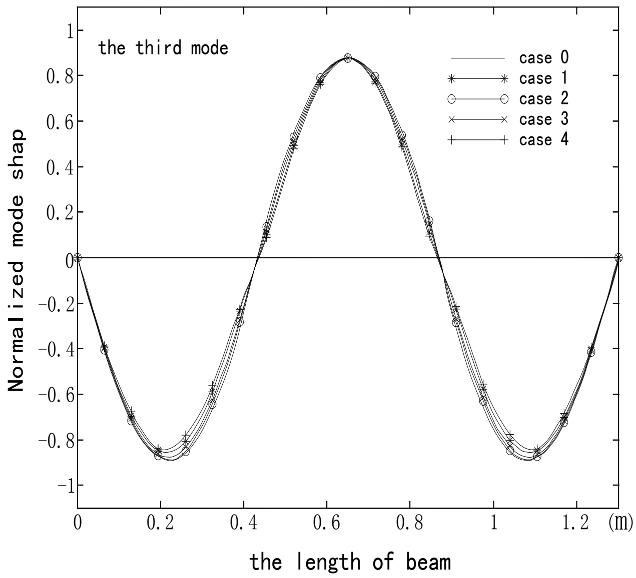


FIG.9 THE THIRD MODE SHAPE FOR SIMULATED BEAM

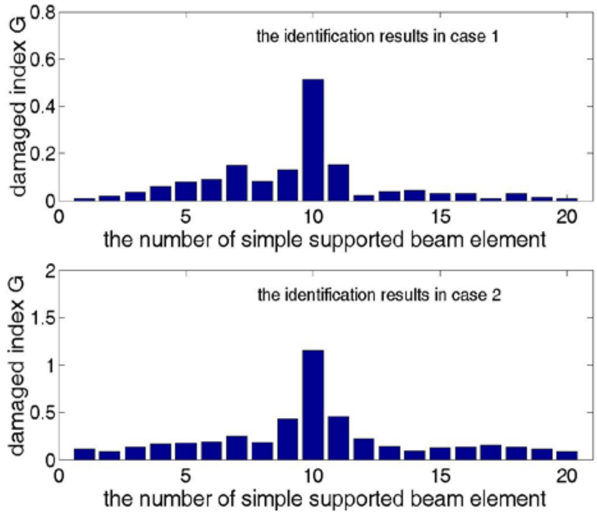


FIG.10 IDENTIFICATION RESULTS FOR SIMULATED BEAM IN CASE 1 AND 2

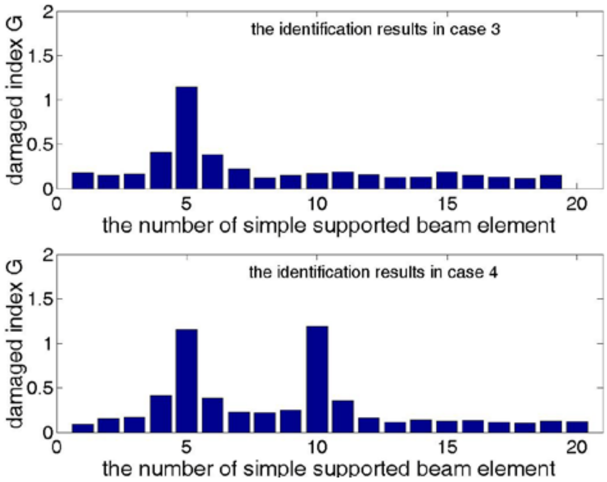


FIG.11 IDENTIFICATION RESULTS FOR SIMULATED BEAM IN CASE 3 AND 4

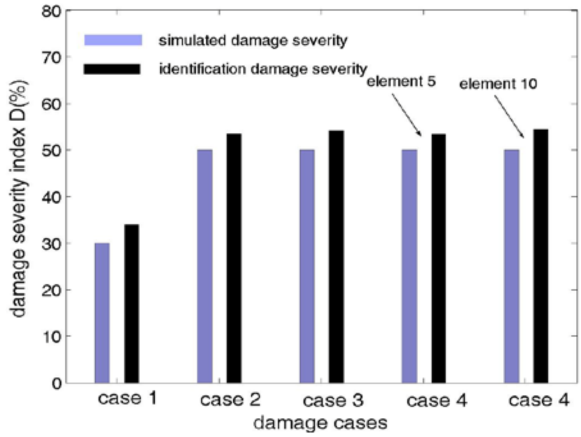


FIG.12 IDENTIFICATION RESULTS OF DAMAGE SEVERITY FOR THE SIMPLE SUPPORTED BEAM

Fig.10~Fig.12 give the damage identification results by using the simulated computation data, which shows again that the VFM method can exactly identify both damage locations and damage severity. All the detection results of damage severity are demonstrated in Fig.12.

Conclusions

The present paper describes a structural damage identification method, called the virtual flexibility matrix(VFM) algorithm, to identify the damage through changes in the dynamic characteristics of two truss model and a simple supported beam. The multiple damage identification results are comparable with structural tests from the other damage identification methods which are always updated or reduced direct stiffness determination. The presented method need not estimate a bending stiffness decrease of the beam element but evaluates the change of the axial displacement for the truss element and of the node's rotation angle for the beam element in undamaged and damaged conditions, respectively. Results of the analysis indicate that the VFM scheme correctly localizes the damage and closely estimates the severity of the damage.

The mode-based VFM technique is based on changes in the structural dynamic characteristics. The dynamic signals obtained from the undamaged and damaged structures are processed by the finite-element package ANSYS. It is feasible to accurately locate damage and estimate damage size in the test structures with the knowledge of as few as three natural frequencies and mode shapes measured before and after damage and with no knowledge of the material properties of the studied structures. Using lower modes extracted from vibration tests can constitute more accurate flexibility matrix and the impacts of numerical matrix (i.e. structural mass matrix) and the material properties of the studied structures in the aforementioned identification procedure could be eliminated from the identification results. However, it is anticipated that the well-designed simulated structural model will be different from actual structures such as bridges and buildings in this studied project. The application of a mode-based VFM technique to a civil engineering structure is of practical interest. It is still a challenge to demonstrate whether damage localization and quantification for a real-life structure can be obtained from the present procedure. Noise, modeling, damage pattern, and numerical stability are still of concern for practical application.

In this study, the uncertainty related to modeling errors, computational errors, or other types were not accounted in this accuracy assessment. Although the represent algorithm shows its robustness in the numerical damaged circumstances, the relative impact of the uncertainties on the accuracy of damage identification using the damage location index G and the damage severity index D will be examined as a extended study and the experimental verification will be discussed in another paper.

ACKNOWLEDGMENTS

This study was supported by the National Natural Science Fundation of China(No.50379029).

REFERENCES

A. K. Pandey, M. Biswas, M. M. Samman, Damage detection from changes in curvature mode shapes, Journal of Sound and Vibration ,145(2) (1991) 321-332.

ANSYS@(1994). User's manual, revision 5.3. Swanson Analysis System.

C. P. Fritzen, D. Jennewein, T. Kiefer, Damage detection based on model updating methods, Mechanical Systems and Signal Processing, 12(1), (1998) 163-168.

Crohas H., and Lepert P.(1982). "Damage detection monitoring method for offshore platforms is field tested." Oil & Gas J., 80(8), 94-103

G.. Hearn, R. B. Testa, Modal analysis for damaged detection in structures, Journal of Structural Engineering, 117(10) (1991), 3042-3063.

H. Y. Hwang, C. Kim, Damage detection in structures using a few frequency response measurements, Journal of Sound and Vibration, 270(2004) 1-14.

J. Cawley, R. D. Adams, Location of defects in structures from measurements of natural frequencies, Journal of Strain Analysis 14(2) (1979) 49-57.

J. Chance, G. R.Tomlinson, Worden K., Simplified approach to the numerical and experimental modeling of the dynamics of a cracked, beam, Proceedings of the 12th International Modal Analysis Conference, Honolulu, CA, 1994, pp.778-785.

J. T. Kim, and N. Stubbs, Model uncertainty impact and damage-detection accuracy in plate girder, Journal of Structural Engineering, 121(10) (1995) 1409-1417.

M. Kaouk, D. C. Zimmerman, Structural damage assessment using a generalized minimum rank perturbation theory, AIAA Journal, 32(4) (1994) 836-842.

MATLAB@(1998). Using MATLAB, revision 5.2. The Mathworks Inc., Natic, MA.

P. Gudmunson, Eigenfrequency changes of structures due to cracks, notches or other geometric changes, Journal of Mechanics and, Physical Solids, 30(1982)., 339-353.

S. Cristides, A. D. S. Barrs, On-orbit damage assessment for large space structures, AIAA Journal 26 (1990) 1119-1126.

W. M. Ostachowicz, M. Krawczuk, Vibration analysis of a cracked beam, Computational Structures 36(2) (1990) 245-250.